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Show that $x^2 + y^2 + z^2 + 2xyz = 1$ has infinitely many integer solutions.

1) $(z+xy)^2 = 1 - x^2 - y^2 + x^2y^2$

$= (x^2-1)(y^2-1)$

let $x^2-1 = 2a^2$ $\exists$ at least one (x,a)
$y^2-1 = 2b^2$ .. (y,b) $\Rightarrow$ infinitely many (x,a) (y,b)

$z+xy = \pm 2ab$

$\therefore \quad z = -xy \pm 2ab$ □

2) let $z=y$

$x^2+y^2+y^2+2xy^2 = 1$

$x^2+2xy^2+2y^2-1 = 0$

$(x+2y^2-1)(x+1) = 0$

$x = -2y^2+1$ Ω